**Positioning and Location Based Services**

**Laboratory 01 – *Reference Systems***

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**Objective**

Compute geodetic coordinates (latitude, longitude and geodetic height) of BRUN, 0001, 0002, 0003 in ETRF at February 2nd, 2019. Compute also the standard deviations of the geodetic coordinates.

**Working Flow**

1. Compute Geocentric Cartesian (GC) coordinates of COMO at 2019/02/02.

They are computed according to the initial position and the velocity given in the webpage of “” and calculated using distributed estimates for permanent stations:

So, replacing in the formula the values for the x, y and z coordinates:

COMOx = 4398306.209 + (-0.0145)\*(2019.09041096 - 2010.0);

COMOy = 704149.948 + 0.0181\*(2019.09041096 - 2010.0);

COMOz = 4550154.733 + 0.0113\*(2019.09041096 - 2010.0);

The results are the following:

1. Compute BRUN coordinates in ITRF.

Having the Como coordinates in ITRF from (1) and knowing the difference between Como and Brunate in ITRF:

1. Compute Local Cartesian coordinates of 0001 with respect to BRUN.

Given a Global Cartesian (GC) known point (BRUN), the rotation matrix can be computed using and coordinates, and with the difference between the known and unknown points in GC, the unknown Local Cartesian (LC) coordinates can be computed.

Where,

P0001\_LC = [ 83.8675 54.4621 1.0108]

1. Convert LC of 0001 to pseudo Local Level with respect to BRUN.

Pseudo Local Level means to compute a LL coordinate tuple only with respect to the X and Y components using the already given and angles (north and east components of vertical deflection), in order to compute the Z rotation matrix.

So, the pseudo or temporary LL coordinates of point 0001 would be:

Where,

xtemp = [83.8675 54.4621 1.0174]

1. Compute alpha rotation between LC and LL in BRUN.

The angle alpha can be computed using the X and Y components of the pseudo LL point 0001:

alpha = 0.5759

Then, the alpha rotation (Z axis rotation) can be computed:

Rot =

0.8387 0.5446 -0.0001

-0.5446 0.8387 -0.0000

0.0000 0.0000 1.0000

1. Compute LC of 0002 and 0003.

Local Level coordinates can be computed using the inverse of the already computed rotation matrix.

P0002\_LC = [ 20.8014 114.8579 1.9933]

P0003\_LC = [ -26.4134 -5.2290 -2.9985]

1. Compute GC of 0002 and 0003.

Having the GC coordinates of BRUN and using the rotation matrix computed in (3), the LC coordinates of 0002 and 0003 can be transformed to GC:

P0001\_GC = [ 4397214.7791890418156981468200684

704153.89153643837198615074157715

4551824.9147216435521841049194336]

P0002\_GC = [ 4397182.6586554059758782386779785

704084.87909476342611014842987061

4551867.7106775762513279914855957]

P0003\_GC = [ 4397271.7245083237066864967346191

704051.32384068646933883428573608

4551780.4391541089862585067749023]

1. Convert through EPN website ITRF GC coordinates of all the stations to ETRF GC coordinates at the same epoch.

The results were the following:

% From ITRF2014 2019.33 to ETRF2014 2019.33

P0001\_GC\_ETRF = [4397215.21430 704153.33680 4551824.58020];

P0002\_GC\_ETRF = [4397183.09380 704084.32430 4551867.37610];

P0003\_GC\_ETRF = [4397272.15960 704050.76910 4551780.10460];

BRUN\_GC\_ETRF = [4397266.34430 704076.58780 4551785.89920];

1. Compute ETRF geodetic coordinates of all the stations.

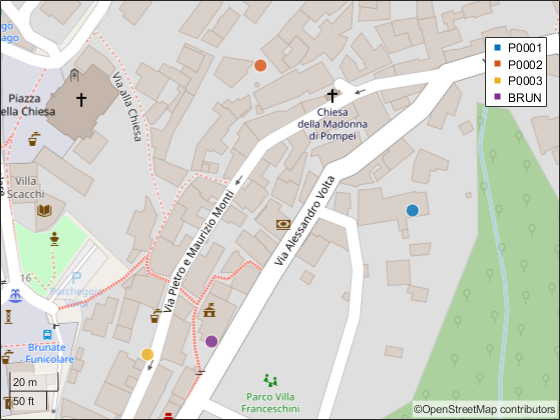
The obtained results were:

P0001\_GEO = [ 45.8196 9.0979 739.1276]

P0002\_GEO = [45.8201 9.0971 740.1104]

P0003\_GEO = [45.8190 9.0965 735.1175]

BRUN\_GEO = [45.8191 9.0968 738.1160]



1. Propagate accuracies from LL in BRUN to LC in BRUN

As it is possible to compute a permanent station position according to initial position, epoch and velocity in ITRF, it is possible to compute its covariance in the same way, for Como:

covComoInd = 1.8264e-06

covComo = 1.0e-05 \*

0.1826 0 0

0 0.1826 0

0 0 0.1826

Given the covariance matrix of Como, it can be propagated to Brunate:

covBrun = 1.0e-05 \*

0.3826 0.0500 0.0500

0.0500 0.2826 0.0500

0.0500 0.0500 0.3826

From As the difference of covariances between Brunate and point 0001, the covariance of the last one can be propagated the same way as before.

cov\_11\_ITRF = 1.0e-05 \*

0.5326 0.0800 0.0800

0.0800 0.3826 0.0700

0.0800 0.0700 0.5826

For the two remaining points 0002 and 0003, a different path must be followed in order to compute their covariances in ITRF. The covariances of the points X, Y and Z components are given for Local Level:

Then, the two covariance matrices will be:

Using the rotation matrix computed in point (6), as for the coordinates, it is possible to transform covariances from Local Level to Local Cartesian:

cov\_22\_LC =

0.0100 0.0000 -0.0000

0.0000 0.0100 -0.0000

-0.0000 -0.0000 0.0225

cov\_33\_LC =

0.0100 0.0000 -0.0000

0.0000 0.0100 -0.0000

-0.0000 -0.0000 0.0225

1. Propagate accuracies from LC in BRUN to ETRF geodetic coordinates

As the Local Cartesian coordinates and covariances are with respect to the Brunate point, it is possible to compute the rotation matrix to transform Local Cartesian to Geocentric Cartesian coordinates as it was done in point (7). But, in this case, we want to obtain the difference of covariances between Brunate and the two points so their covariances can be propagated as before.

RotLC2GC =

-0.1581 0.9874 0

-0.7081 -0.1134 0.6969

0.6882 0.1102 0.7171

deltaBrun\_22 =

0.0159 0.0009 0.0062

0.0009 0.0102 0.0010

0.0062 0.0010 0.0164

cov\_22\_ITRF =

0.0159 0.0009 0.0062

0.0009 0.0102 0.0010

0.0062 0.0010 0.0164

deltaBrun\_33 =

0.0159 0.0009 0.0062

0.0009 0.0102 0.0010

0.0062 0.0010 0.0164

cov\_33\_ITRF =

0.0159 0.0009 0.0062

0.0009 0.0102 0.0010

0.0062 0.0010 0.0164

Having all the covariances in ITRF (geocentric), they can be propagated to local baselines:

cov\_Brun\_ENU =

1.0e-05 \*

0.2695 0.0060 0.0520

0.0060 0.3321 -0.0082

0.0520 -0.0082 0.4463

And so, the standard deviations ENU for points 0001, 0002 and 0003 can be computed:

SIGMA\_E\_11 =

0.0019

SIGMA\_N\_11 =

0.0022

SIGMA\_U\_11 =

0.0026

 SIGMA\_E\_22 =

0.1000

 SIGMA\_N\_22 =

0.1000

 SIGMA\_U\_22 =

0.1500

 SIGMA\_E\_33 =

0.1000

SIGMA\_N\_33 =

0.1000

 SIGMA\_U\_33 =

0.1500